

# On the finite-sample size distortion of smooth transition unit root tests

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Received 1 June 2004

Available online 4 November 2004

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## Abstract

The finite-sample size properties of smooth transition unit root tests are examined when applied to unit root processes subject to breaks in either level or drift. In contrast to the weighted symmetric and recursively mean-adjusted unit root tests which have been shown to be robust in these circumstances, it is found that the empirical sizes of smooth transition tests are dependent upon the form, location and magnitude of the break imposed. It is concluded that while smooth transition unit root tests are capable of capturing breaks under an alternative hypothesis of stationarity, spurious rejection can occur when breaks occur under the null.

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*Keywords:* Unit roots; Smooth transitions; Structural change

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## 1. Introduction

Examination of the integrated nature of time series data is a familiar feature of applied research in econometrics and time series analysis. However, following Perron (1989), it has long been recognised that the frequently employed Dickey and Fuller (1979) (DF) unit root test can exhibit low power when applied to a series which is stationary about a deterministic component subject to structural change. In response to this finding, a large literature has emerged allowing the unit root

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hypothesis to be tested in the presence of structural change. While some authors have allowed for the presence of instantaneous change which is either exogenously or endogenously determined, [Leybourne, Newbold and Vougas \(1998\)](#) (LNV) suggest an alternative procedure which allows for gradual adjustment.<sup>1</sup> The smooth transition unit root tests of LNV employ the logistic smooth transition function to permit testing of the unit root hypothesis against an alternative of stationarity with structural change in the form of a gradual adjustment between two regimes. As LNV note, such an approach has an obvious intuitive appeal, particularly in the analysis of economics data. However, while the properties of the smooth transition unit root tests of LNV and the subsequent test of [Vougas \(2004\)](#) have been examined in the presence of structural change under the alternative hypothesis of stationarity, the behaviour of the tests are as yet unknown when structural change occurs under the null. Following the research of [Leybourne, Mills and Newbold \(1998\)](#) (LMN), a ‘converse Perron phenomenon’ has been noted in the literature, with the DF test exhibiting severe size distortion when applied to unit root processes subject to a break in either level or drift. The resulting spurious rejection of the unit root null hypothesis in such circumstances can, therefore, lead the practitioner to falsely conclude an  $I(1)$  series with a break is  $I(0)$ . In the present paper, it is examined whether smooth transition unit root tests are subject to similar size distortion when either breaks in level or trend occur under the null.

## 2. Smooth transition unit root tests

LNV introduce three smooth transition unit root tests based upon the following models denoted as A, B and C:

$$\text{Model A: } y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + u_{at},$$

$$\text{Model B: } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + u_{bt},$$

$$\text{Model C: } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 t S_t(\gamma, \tau) + u_{ct},$$

where  $u_{it}$  are zero mean  $I(0)$  error processes.  $S_t(\gamma, \tau)$  is a deterministic logistic smooth transition trend function, which, for a sample of  $T$  observations, is defined as

$$S_t(\gamma, \tau) = [1 + \exp\{-\gamma(t - \tau T)\}]^{-1}, \quad \gamma > 0, \quad t = 1, \dots, T, \quad (1)$$

where the parameter  $\tau$  determines the fraction of the sample at which the transition occurs, while  $\gamma$  determines the speed of transition. Considering Model A,  $y_t$  is stationary about a mean which changes in value from  $\alpha_1$  to  $(\alpha_1 + \alpha_2)$  at time  $\tau T$ . Model B extends this specification to allow for the presence for a linear trend, while Model C permits a change in both intercept and trend, with the slope of the trend changing from  $\beta_1$  to  $(\beta_1 + \beta_2)$ . In more recent research, [Vougas \(2004\)](#) proposes a further specification denoted as Model D which permits a change in trend only

$$\text{Model D: } y_t = \alpha_1 + \beta_2 t S_t(\gamma, \tau) + u_{dt}.$$

<sup>1</sup>The research of [Perron \(1989\)](#) and [Zivot and Andrews \(1992\)](#) provide examples of studies where instantaneous structural change is determined exogenously and endogenously.

To test for the presence of a unit root, the null hypothesis of a unit root or a unit root with drift is tested against an alternative given by Model A, B, C or D as appropriate. In each case, a two-step approach is followed.<sup>2</sup> In the first step, the appropriate specification above (Model A, B, C or D) is estimated using a nonlinear least squares (NLS) algorithm and the resulting residuals ( $\hat{u}_{it}$ ,  $i = a, b, c, d$ ) are stored. In the second step, an augmented DF test is performed using the  $t$ -ratio of  $\psi_i$  from the following regression:

$$\Delta \hat{u}_{it} = \psi_i \hat{u}_{it-1} + \sum_{j=1}^{p_i} \phi_{ij} \Delta \hat{u}_{it-j} + \varepsilon_{it}, \quad i = a, b, c, d. \quad (2)$$

While the properties of the above tests have been considered via simulation analysis of power and empirical application to the macroeconomics data of Nelson and Plosser (1982), the behaviour of the tests in the presence of breaks under the null have yet to be established. In the following sections the properties of smooth transition unit root tests are considered when either a break in level or drift occurs under the null.

### 3. Breaks in level

#### 3.1. Experimental design

In this section the behaviour of the unit root tests are examined in the presence of level breaks. Given the break in level and absence of a trend, the smooth transition unit root test considered is based upon the use of Model A. This test is denoted as  $s_\lambda$ . In addition, for purposes of comparison, the empirical size of the DF  $\tau_\mu$  test is also calculated. To generate unit root processes subject to a break in level, the following experimental design of LMN is employed:

$$y_t = \delta I_t(\lambda) + \xi_t, \quad t = 1, \dots, T, \quad (3)$$

$$\xi_t = \xi_{t-1} + \eta_t, \quad (4)$$

$$\eta_t \sim \text{i.i.d. } N(0, 1), \quad (5)$$

$$I_t(\lambda) = \begin{cases} 0 & \text{for } t \leq \lambda T, \\ 1 & \text{for } t > \lambda T, \end{cases} \quad \lambda \in (0, 1). \quad (6)$$

The error series  $\{\eta_t\}$  is generated using the RNDNS procedure in the Gauss programming language. Due to the computationally intensive nature of  $s_\lambda$ , all experiments are performed over

<sup>2</sup>In the interest of brevity calculation of the smooth transition unit root tests is only outlined here. Further details can be obtained from reference to LNV and Vougas (2004).

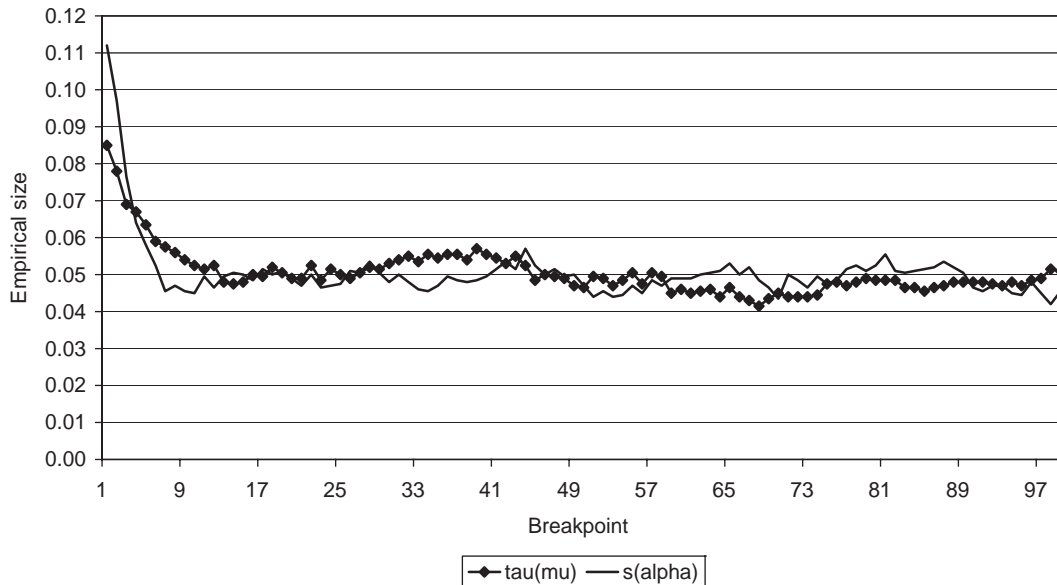


Fig. 1. Empirical size in the presence of level breaks ( $\Delta = 2.5$ )

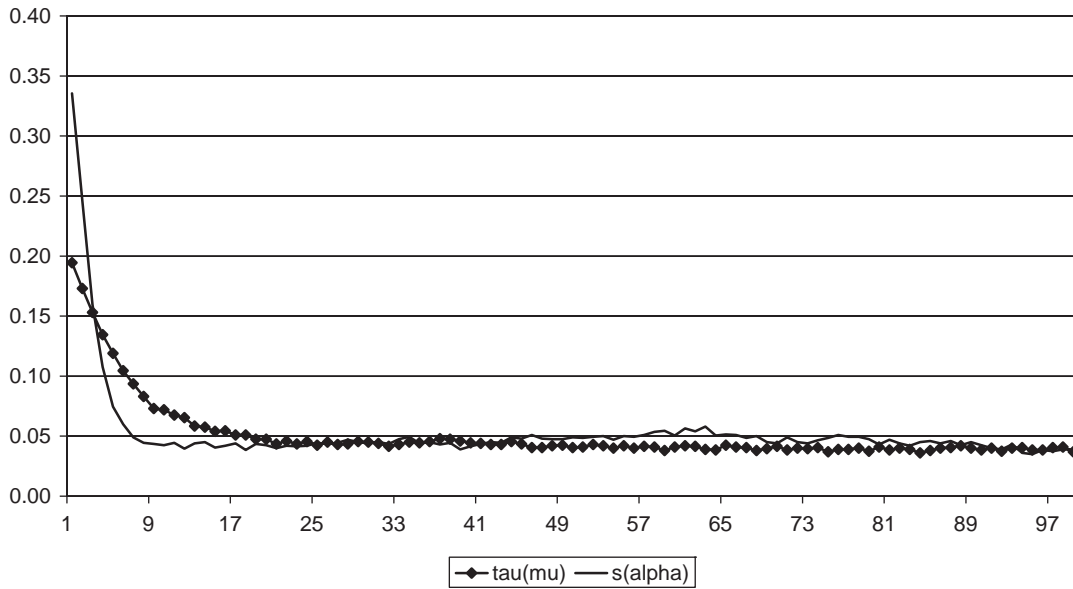
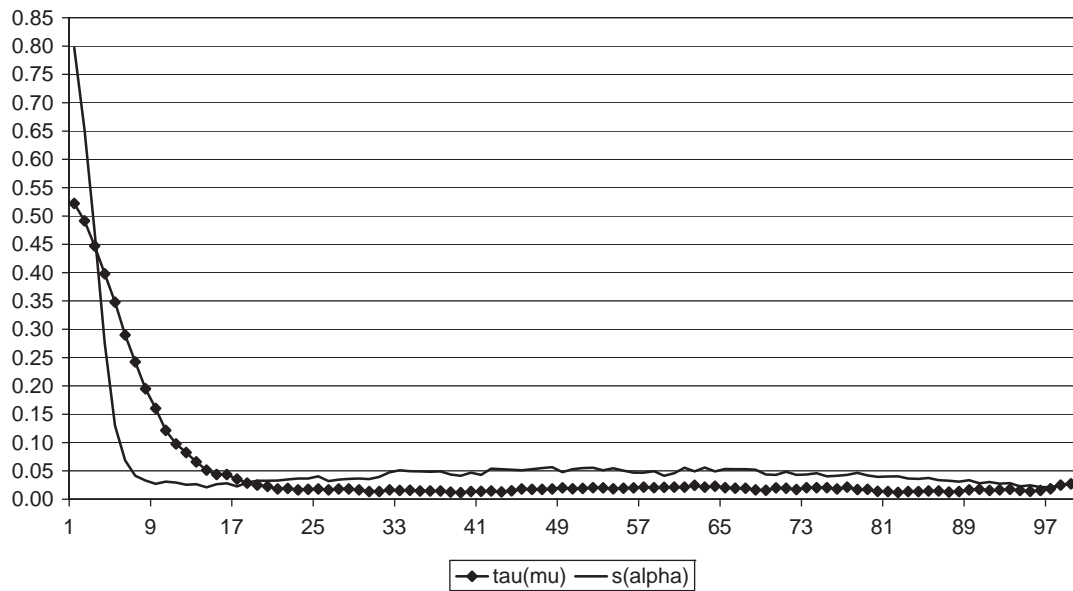
2000 replications. Following LMN a sample size of 100 observations is employed, with  $\xi_0 = 0$ .<sup>3</sup> To further replicate the experimental design of LMN, the values  $\delta \in \{2.5, 5, 10\}$  are selected for the break magnitude. Denoting the break fraction as  $\lambda$ , where  $\lambda = \{0.01, 0.02, \dots, 0.99\}$ , the break in level is imposed after observation  $\lambda T$ . The (false) rejections of the unit root hypothesis are noted at the 5% level of significance using the corrected critical values provided by Vougas (2004) for the  $s_\alpha$  test and Dickey and Fuller (1979) for the  $\tau_\mu$  test.<sup>4</sup>

### 3.2. Experimental results

To ease interpretation, the experimental results obtained are presented graphically in Figs. 1–3. From inspection of these graphs, it is apparent that both the  $s_\alpha$  and  $\tau_\mu$  tests exhibit oversizing when breaks occur early in the sample period and that oversizing is greater in the presence of larger breaks. Considering the case of the largest break of  $\delta = 10$  reported in Fig. 3, the maximum empirical sizes observed are 79.7% for  $s_\alpha$  and 52.5% for  $\tau_\mu$  when  $\lambda = 0.01$ . The corresponding maximum rejection frequencies for the smallest break of  $\delta = 2.5$  are 11.2% and 8.5%. However, while the maximum size of  $s_\alpha$  is greater than that of  $\tau_\mu$ ,  $s_\alpha$  exhibits oversizing over a smaller range

<sup>3</sup>Further similar results for alternative sample sizes are available from the authors upon request.

<sup>4</sup>Vougas (2004) provides a detailed discussion of alternative approaches to the NLS estimation required for the above smooth transition unit root tests, with close attention paid to the impact of differing optimisation algorithms upon resulting critical values. In particular, revised critical values are provided for the smooth transition tests employing Models A, B and C using the superior NLP<sup>®</sup>-constrained optimiser of the GAUSS subroutine FANPAC<sup>®</sup>. This optimiser combines the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithms utilised by LNV, with the Newton–Raphson algorithm. In this paper, the arguments of Vougas (2004) are followed, with the revised critical values employed and the NLP<sup>®</sup>-constrained optimiser employed to estimate the required smooth transition tests.

Fig. 2. Empirical size in the presence of level breaks ( $\Delta = 5$ )Fig. 3. Empirical size in the presence of level breaks ( $\Delta = 10$ )

of breakpoints. In addition,  $s_x$  does not exhibit the undersizing noted for  $\tau_\mu$ , particularly in the presence of the largest break where a minimum size of 1.15% is observed for  $\tau_\mu$  when  $(\delta, \lambda) = (10, 0.39)$ .

#### 4. Breaks in drift

##### 4.1. Experimental design

To analyse the break in drift case, the earlier DGP of (3)–(6) is modified as below<sup>5</sup>

$$y_t = \delta I_t(\lambda) + y_{t-1} + \xi_t, \quad t = 1, \dots, T, \quad (7)$$

$$\xi_t \sim \text{i.i.d. } N(0, 1), \quad (8)$$

$$I_t(\lambda) = \begin{cases} 0 & \text{for } t \leq \lambda T, \\ 1 & \text{for } t > \lambda T, \end{cases} \quad \lambda \in (0, 1). \quad (9)$$

For this experimental design with a break in drift, the appropriate specifications of the unit root tests to consider are the smooth transition unit root test based upon Model D, denoted as  $s_\beta$ , and the DF  $\tau_\tau$  test. Again, breaks are imposed after observation  $\lambda T$  with the following break magnitudes considered:  $\delta \in \{0.5, 1, 2\}$ . False rejections of the null are noted at the 5% level of significance using the critical values of Vougas (2004) and Dickey and Fuller (1979), respectively.

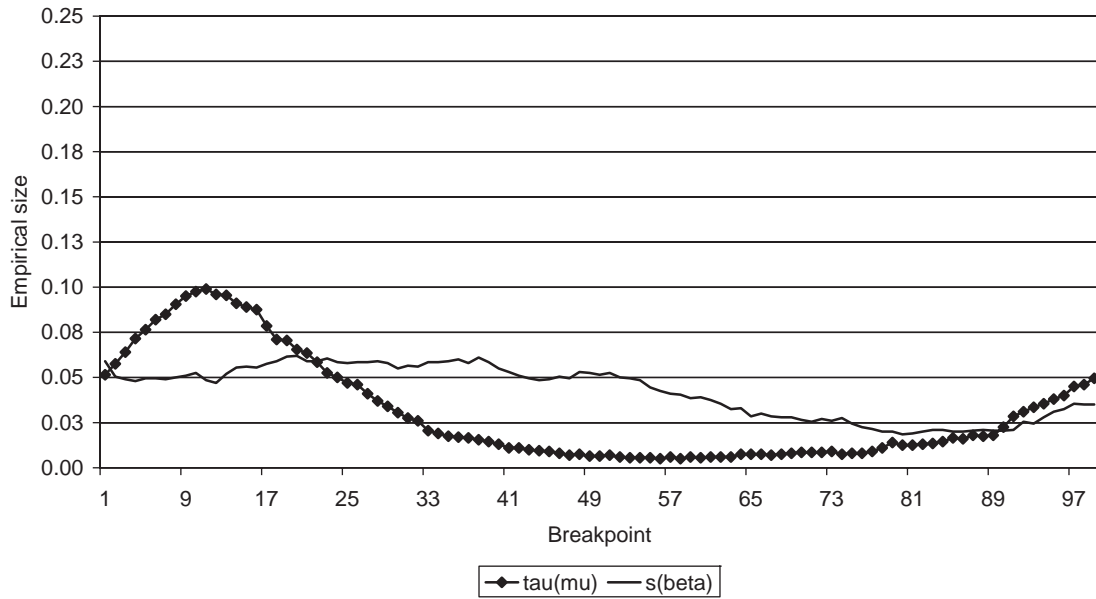
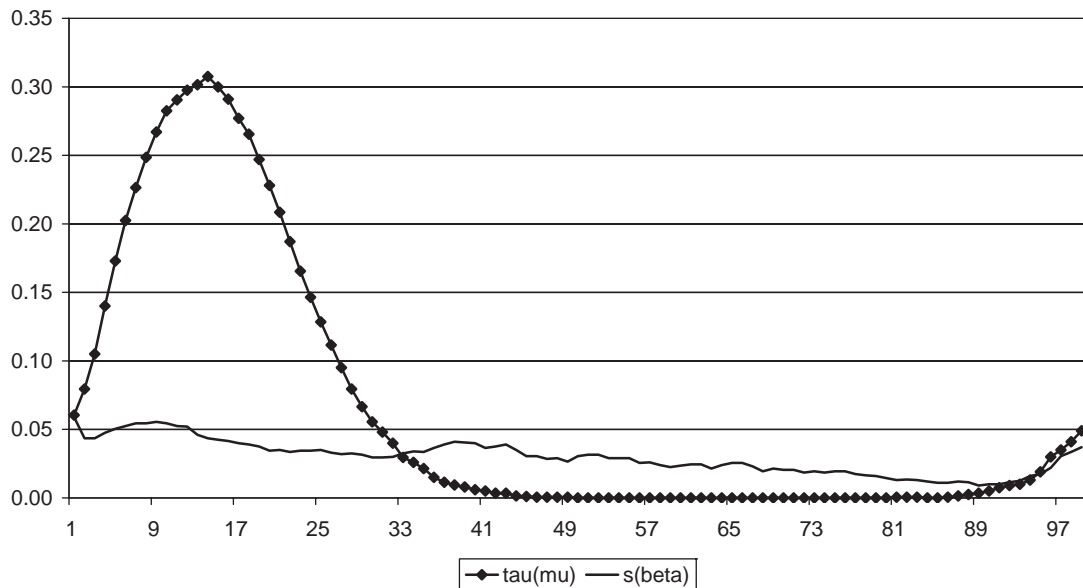
##### 4.2. Experimental results

The empirical sizes of  $s_\beta$  and  $\tau_\mu$  in the presence of breaks in drift of alternative sizes are presented in Figs. 4–6. Considering the results for  $\tau_\tau$ , it is apparent that the empirical size of the test is highly dependent upon the timing of the break. While breaks in the early part of the sample can generate severe oversizing, with a maximum size of 93.5% observed when  $(\delta, \lambda) = (2, 0.16)$ , later breaks are seen to result in severe undersizing with an empirical size of 0 noted over a range of breakpoints in Figs. 5 and 6. In contrast, the smooth transition test,  $s_\beta$ , appears more robust to breaks in drift. To illustrate this point, consider the maximum empirical sizes of 11.3% ( $s_\beta$ ) and 93.8% ( $\tau_\tau$ ) obtained in the presence of the largest break in drift ( $\delta = 2$ ). In addition, the  $s_\beta$  exhibits less pronounced undersizing for breaks occurring later in the sample period.

#### 5. Conclusion

In this paper, the finite-sample size properties of smooth transition unit root tests have been examined in the presence of a structural change under the null. While the properties of the tests in the presence of breaks under the alternative have been established, their behaviour when applied to unit root processes subject to structural change had not been considered previously. The experimental results obtained showed the empirical size of the tests to depend upon the magnitude, timing and type of break considered. While large breaks in level at the start of the sample period were found to cause severe oversizing in the smooth transition unit root test  $s_\alpha$ , with the observed size distortion exceeding that of the D–F unit root test, the test was otherwise relatively robust to level breaks. When considering breaks in drift, the smooth transition unit root

<sup>5</sup>The treatment of initial conditions, method of random number generation, sample size, and number of replications and discards for the break in drift experiments are the same as for the earlier level break experiments.

Fig. 4. Empirical size in the presence of breaks in drift ( $\Delta = 0.5$ )Fig. 5. Empirical size in the presence of breaks in drift ( $\Delta = 1$ )

test  $s_\beta$  exhibited both oversizing and undersizing in the presence of the largest break considered depending upon whether the break was imposed relatively early or late in the sample period. For smaller breaks in drift, moderate undersizing was observed. In summary, the results obtained

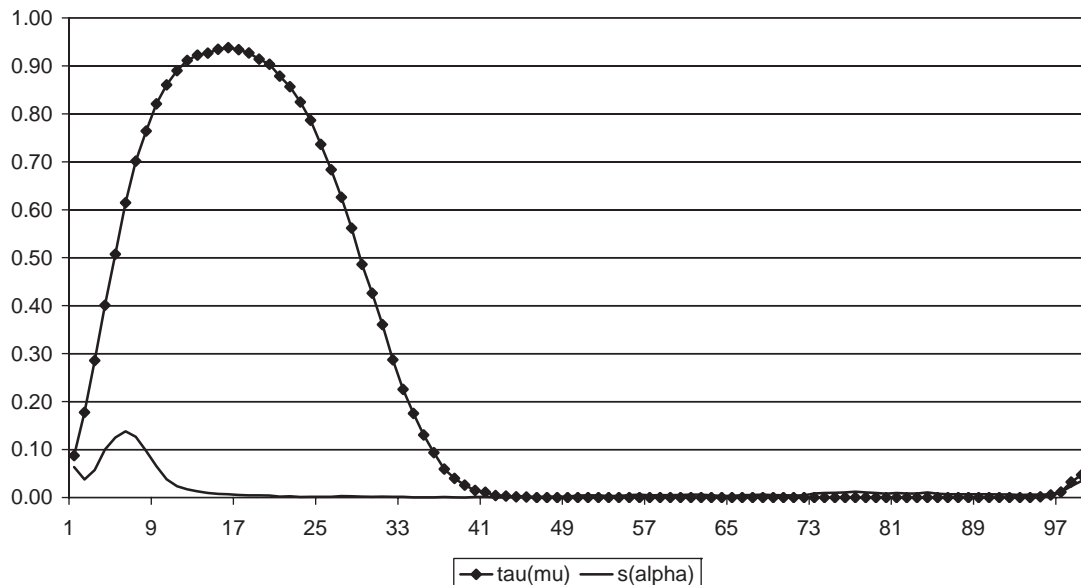


Fig. 6. Empirical size in the presence of breaks in drift ( $\Delta=2$ )

showed that while the application of smooth transition unit root tests can aid detection of stationarity in the presence of structural change, it is possible that this may be spurious as a break exists under the null. In this regard, smooth transition unit root tests do not possess the robustness previously noted for weighted symmetric and recursively mean-adjusted D–F tests in the presence of breaks under the null (see [Leybourne and Newbold, 2000](#); [Cook, 2002](#)).

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